



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\pm \frac{x\sqrt{[1+(dx/dy)^2 + (dx/dz)^2]}}{y\sqrt{[1+(dy/dx)^2 + (dy/dz)^2]}} = a = \pm \frac{xdx}{ydy} \dots (1).$$

$$\pm \frac{x\sqrt{[1+(dx/dy)^2 + (dx/dz)^2]}}{z\sqrt{[1+(dz/dx)^2 + (dz/dy)^2]}} = b = \pm \frac{xdx}{zdz} \dots (2).$$

$$\pm \frac{y\sqrt{[1+(dy/dx)^2 + (dy/dz)^2]}}{z\sqrt{[1+(dz/dx)^2 + (dz/dy)^2]}} = c = \pm \frac{ydy}{zdz} \dots (3).$$

$\therefore aydy = xdx, bzdz = xdx, czdz = ydy, aydy + bzdz + czdz = 2xdx + ydy.$

$$\therefore 2x^2 = (b+c)z^2 + (a-1)y^2 + D.$$

$\therefore x^2 = Ay^2 + Bz^2 + D$  are the surfaces satisfying the conditions.

The equation could be put in the form  $x^2 = ay^2 - acz^2 + D = 0$ , where  $ac = -b$ , as is the case from (1), (2), and (3).

323. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

$S, S'$  are the foci of two co-vertical parabolas  $A$  and  $B$ , the axes of which are at right angles. Draw the circle  $K$  on  $SS'$  as diameter.  $K$  is cut in  $D$  and  $E$  by a straight line parallel to the axis of  $A$  such that  $S'$  lies midway between it and that axis. Show that the lines  $S'D, S'E$  are parallel to the two tangents to  $A$  which are normals to  $B$ .

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

The problem as stated is not true.

Let  $y^2 = 4ax, x^2 = 4by$ , be the parabolas  $A, B$ ;  $y = mx + a/m =$  tangent to  $A; y - y' = m(x - x')$  the normal to  $B$ .

Since  $x'^2 = 4by', m = -2b/x'$  or  $x' = -2b/m, y' = x'^2/4b = b/m^2$ .

$\therefore y = mx + 2b + b/m^2 =$  normal to  $B$ .

But  $y = mx + a/m$  and  $y = mx + 2b + b/m^2$  are the same line.

$\therefore a/m = 2b + b/m^2$ .

$\therefore m = \frac{1}{4b} [a \pm \sqrt{(a^2 - 8b^2)}]$ , which is true for  $a \geq 2\sqrt{2}b$ .

$x^2 - ax + y^2 - by = 0$  is equation to  $K, y = 2b$  = line parallel to axis of  $A$ .

$\therefore x^2 - ax + 2b^2 = 0$  or  $x = \frac{1}{2}[a \pm \sqrt{(a^2 - 8b^2)}]$ .

$\therefore y = \frac{2bx}{a \pm \sqrt{(a^2 - 8b^2)}} + b$  are the equations to  $S'D, S'E$ .

$\therefore$  If  $m$  were twice as great,  $S'D, S'E$  would be perpendicular to the two tangents to  $A$  which are normals to  $B$ .

The lines through  $S'$  parallel to the two tangents are given by the equation

$$y = \frac{1}{4b} [a \pm \sqrt{(a^2 - 8b^2)}]x + b.$$

This line intersects  $K$  in the points

$$\left[ -\frac{6ab^2 \pm 2b^2 \sqrt{(a^2 - 8b^2)}}{a^2 + 4b^2 \pm a\sqrt{(a^2 - 8b^2)}}, -\frac{a^2b - 12b^3 \pm ab\sqrt{(a^2 - 8b^2)}}{a^2 + 4b^2 \pm a\sqrt{(a^2 - 8b^2)}} \right],$$

the plus and minus signs to be used together.

$$y = \frac{8abx}{a^2 + 4b^2} + 12b^3 - a^2b + \frac{48a^2b^3}{a^2 + 4b^2 + a\sqrt{(a^2 - 8b^2)}}$$

is the line through these points. The tangents to  $A$  parallel to  $S'D$ ,  $S'E$  are

$$y = \frac{2bx}{a \pm \sqrt{(a^2 - 8b^2)}} + \frac{a[a \pm \sqrt{(a^2 - 8b^2)}]}{2b}$$

This line meets  $x^2 = 4by$  in

$$x_1 = \frac{4b^2 \pm \sqrt{16b^4 + 2a[a \pm \sqrt{(a^2 - 8b^2)}]^3}}{a \pm \sqrt{(a^2 - 8b^2)}} = r.$$

The tangent at this point makes an angle with the axis of abscissas whose tangent is  $x_1/2b$ . As this does not equal  $-1/m$  the problem, as stated, is not true.

### CALCULUS.

252. Proposed by J. H. MEYER, S. J., Augusta, Ga.

Supposing the arc of a semi-circle to be stretched out into a straight line, and an indefinite number of perpendiculars erected on it, each equal to the versed sine of the corresponding arc; what would be the length of the curve traced out by the tops of the perpendiculars?

**Solution by CHAS. O. GUNTHER, Acting Professor of Mathematics, Stevens Institute of Technology, Hoboken, N. J.**

Assuming  $a$  as the radius of the circle, the equation of the curve is  $y = \text{vers } x/a$ , and the required length of the curve is given by the expression

$$s = 2 \int_0^{\pi a/2} \left(1 + \frac{\sin^2 x/a}{a^2}\right) dx = 2 \int_0^{\pi a/2} \sqrt{a^2 + 1 - \cos^2 x/a} \frac{dx}{a}.$$

Let  $\cos x/a = \sin \theta$ ; then

$$s = 2 \sqrt{a^2 + 1} \int_0^{\frac{1}{2}\pi} \sqrt{1 - \left(\frac{1}{\sqrt{a^2 + 1}}\right)^2 \sin^2 \theta} d\theta = 2 \sqrt{a^2 + 1} E\left(\frac{1}{\sqrt{a^2 + 1}}, \frac{1}{2}\pi\right).$$

Also solved by G. B. M. Zerr.